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Note on "roughness in Cayley graphs"

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Abstract

In this note, we show that some results in [M.H. Shahzamanian, M. Shirmohammadi and B. Davvaz, Roughness in Cayley graphs, Information Sciences, 180, 2010, 3362-3372] are not correct and present their modified versions.

Keywords: Cayley graph, Pseudo-Cayley graph, Rough set, Lower and upper approximations.

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1. Introduction

Rough set theory was proposed by Pawlak as an extension of set theory [6]. In 1996, Kuroki and Wang introduced the notion of a rough set with respect to a normal subgroup of a group and investigated some properties of the lower and the upper approximations in a group [5]. But a decade later, Cheng et al. proved that Propositions 2.4 and 2.5 of [5] are incorrect and gave their updated versions.

The Cayley graphs are the popular representations of groups by graphs, where first studied by Cayley [2, 3]. In [7], Shahzamanian et al. studied rough approximations of Cayley graphs and pseudo-Cayley graphs. They derived Theorems 4.6, 5.6 and 6.6 by using of Propositions 2.4 and 2.5 of [5]. The aim of this note is to offer the modified versions of these incorrect theorems in [7].

2. Preliminaries

In the following, we first briefly review some definitions and terminologies related to rough sets and graphs. For rough set and graph-theoretic concepts not defined here, we refer to [5] and [1], respectively. In this note, all groups and graphs are finite.

Throughout this note, let $D_{2n} = \langle a, b \mid a^n = b^2 = 1,\ b^{-1}ab = a^{-1} \rangle$ be the dihedral group of order $2n$, $n \geqslant 1$.

Let G be a group with identity 1, N be a normal subgroup of G and A be a non-empty subset of G. Then the sets $N_-(A) = \{x \in G \mid Nx \subseteq A\}$ and $N^\wedge(A) = \{x \in G \mid Nx \cap A \neq \emptyset\}$ are called, respectively, the lower and upper approximations of A with respect to N. Also, $N(A) = \big(N_-(A), N^\wedge(A)\big)$ is called a rough set of A in G.

The statements of the following proposition are the modified versions of Propositions 2.4 and 2.5 in [5].

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Proposition 2.1 ([4]). Let H and N be normal subgroups of a group G. Let A be a non-empty subset of G. Then

(1) $(H\cap N)_{-}(A) \supseteq\ H_{-}(A)\cup N_{-}(A)\ \supseteq H_{-}(A)\cap N_{-}(A)$,

(2) $(H\cap N)^{\wedge}(A) \subseteq\ H^{\wedge}(A)\cap N^{\wedge}(A)\ \subseteq H^{\wedge}(A)\cup N^{\wedge}(A)$.

The union $\Gamma_1\cup\Gamma_2$ of two graphs Γ_1 and Γ_2 is a graph with vertex set $V(\Gamma_1)\cup V(\Gamma_2)$ and edge set $E(\Gamma_1)\cup E(\Gamma_2)$. The intersection $\Gamma_1\cap\Gamma_2$ of Γ_1 and Γ_2 is defined analogously.

Let S be a subset of a group G such that $1\notin S$ and $S=S^{-1}$. The Cayley graph $Cay(G,S)$ is a graph with vertex set G and edge set $\{\{g,gs\}|\ g\in G,\ \ s\in S\}$.

Definition 2.2 ([7]). Let S be a subset of a group G such that $1\notin S$ and $S=S^{-1}$. Let R be a subset of G such that $S\subseteq R$ and $RS\subseteq R$, where $RS=\{rs|\ r\in R, s\in S\}$. The pseudo-Cayley graph $PCay(R,S)$ is a graph with vertex set R and edge set $\{\{r,rs\}|\ r\in R, s\in S\}$.

Definition 2.3 ([7]). Let N be a normal subgroup of a group G and $\Gamma=Cay(G,S)$ be a Cayley graph. Then $\underline{\Gamma}=Cay(G,N_{-}(S))$ and $\overline{\Gamma}=Cay(G,N^{\wedge}(S)^{*})$, where $N^{\wedge}(S)^{*}=N^{\wedge}(S)-1$, are called, respectively, the lower and upper approximations edge Cayley graphs of Γ with respect to N. Also, $(\underline{\Gamma},\overline{\Gamma})$ is called a rough edge Cayley graph of Γ.

Definition 2.4 ([7]). Let R be a subset of a group G, N be a normal subgroup of G and $\Gamma=PCay(R,S)$ be a pseudo-Cayley graph. Then $\underline{\Gamma}'=PCay(N_{-}(R),S\cap N_{-}(R))$ and $\overline{\Gamma}'=PCay(N^{\wedge}(R),S)$, are called, respectively, the lower and upper approximations vertex pseudo-Cayley graphs of Γ with respect to N. Also, $(\underline{\Gamma}',\overline{\Gamma}')$ is called a rough vertex pseudo-Cayley graph of Γ.

Definition 2.5 ([7]). Let R be a subset of a group G, N be a normal subgroup of G and $\Gamma=PCay(R,S)$ be a pseudo-Cayley graph. Then $\underline{\Gamma}''=PCay(N_{-}(R),N_{-}(S))$ and $\overline{\Gamma}''=PCay(N^{\wedge}(R),N^{\wedge}(S)^{*})$, are called, respectively, the lower and upper approximations pseudo-Cayley graphs of Γ with respect to N. Also, $(\underline{\Gamma}'',\overline{\Gamma}'')$ is called a rough pseudo-Cayley graph of Γ.

3. Counterexamples

Theorem 4.6 in [7] is as follows:

Let N and H be normal subgroups of a group G and $\Gamma=Cay(G,S)$ be a Cayley graph. Then $\underline{\Gamma}_{H\cap N}=\underline{\Gamma}_H\cap\underline{\Gamma}_N$ and $\overline{\Gamma}_{H\cap N}=\overline{\Gamma}_H\cap\overline{\Gamma}_N$.

The following example shows that both $\underline{\Gamma}_{H\cap N}\subseteq\underline{\Gamma}_H\cap\underline{\Gamma}_N$ and $\overline{\Gamma}_{H\cap N}\supseteq\overline{\Gamma}_H\cap\overline{\Gamma}_N$ do not hold in general.

Example 3.1. Let $G=\mathbb{Z}_2\times\mathbb{Z}_2=\langle a\rangle\times\langle b\rangle, N=\{1,a\}, H=\{1,b\}, S=\{a,b\}$ and $\Gamma=Cay(G,S)$. Then $N_{-}(S)=H_{-}(S)=\emptyset$ and $(H\cap N)_{-}(S)=\{a,b\}$. It is obvious that $\underline{\Gamma}_{H\cap N}\nsubseteq\underline{\Gamma}_H\cap\underline{\Gamma}_N$. Also, $N^{\wedge}(S)^{*}=H^{\wedge}(S)^{*}=\{a,b,ab\}$ and $(H\cap N)^{\wedge}(S)^{*}=\{a,b\}$. It is clear that $\overline{\Gamma}_{H\cap N}\nsupseteq\overline{\Gamma}_H\cap\overline{\Gamma}_N$.

Theorem 5.6 in [7] is as follows:

Let N and H be normal subgroups of a group G and $\Gamma=PCay(R,S)$ be a pseudo-Cayley graph. Then $\underline{\Gamma}'_{H\cap N}=\underline{\Gamma}'_H\cap\underline{\Gamma}'_N$ and $\overline{\Gamma}'_{H\cap N}=\overline{\Gamma}'_H\cap\overline{\Gamma}'_N$.

The following example shows that both $\underline{\Gamma}'_{H\cap N}\subseteq\underline{\Gamma}'_H\cap\underline{\Gamma}'_N$ and $\overline{\Gamma}'_{H\cap N}\supseteq\overline{\Gamma}'_H\cap\overline{\Gamma}'_N$ are not necessarily true.

Example 3.2. Let $G=\mathbb{Z}_2\times\mathbb{Z}_2=\langle a\rangle\times\langle b\rangle, R=\{1,ab\}, N=\{1,a\}, H=\{1,b\}, S=\{ab\}$ and $\Gamma=PCay(R,S)$. Then $N_{-}(R)=H_{-}(R)=\emptyset$ and $(H\cap N)_{-}(R)=\{1,ab\}$. Obviously $\underline{\Gamma}'_{H\cap N}\nsubseteq\underline{\Gamma}'_H\cap\underline{\Gamma}'_N$. Also, $N^{\wedge}(R)=H^{\wedge}(R)=\{1,a,b,ab\}$ and $(H\cap N)^{\wedge}(R)=\{1,ab\}$. So, $\overline{\Gamma}'_{H\cap N}\nsupseteq\overline{\Gamma}'_H\cap\overline{\Gamma}'_N$.

Theorem 6.6 in [7] is as follows:

Let N and H be normal subgroups of a group G and $\Gamma = PCay(G, S)$ be a pseudo-Cayley graph. Then $\underline{\Gamma}''_{H\cap N} = \underline{\Gamma}''_H \cap \underline{\Gamma}''_N$ and $\overline{\Gamma}''_{H\cap N} = \overline{\Gamma}''_H \cap \overline{\Gamma}''_N$.

The following example shows that both $\underline{\Gamma}''_{H\cap N} \subseteq \underline{\Gamma}''_H \cap \underline{\Gamma}''_N$ and $\overline{\Gamma}''_{H\cap N} \supseteq \overline{\Gamma}''_H \cap \overline{\Gamma}''_N$, however, are not true.

Example 3.3.

(1) Let $G = \mathbb{Z}_{12}, R = \{0, 2, 4, 5, 6, 8, 10, 11\}, N = \{0, 4, 8\}, H = \{0, 6\}, S = \{6\}$ and $\Gamma = PCay(R, S)$. Then $N_-(R) = \{0, 2, 4, 6, 8, 10\}$ and $H_-(R) = (H \cap N)_-(R) = \{0, 2, 4, 5, 6, 8, 10, 11\}$. Also, $N_-(S) = H_-(S) = \emptyset$ and $(H \cap N)_-(S) = \{6\}$. Obviously, $\underline{\Gamma}''_{H\cap N} \nsubseteq \underline{\Gamma}''_H \cap \underline{\Gamma}''_N$.

(2) Let $G = \mathbb{Z}_2 \times \mathbb{Z}_2 = \langle a\rangle \times \langle b\rangle, R = \{1, ab\}, N = \{1, a\}, H = \{1, b\}, S = \{ab\}$ and $\Gamma = PCay(R, S)$. Then $N^\wedge(R) = H^\wedge(R) = \{1, a, b, ab\}$ and $(H \cap N)^\wedge(R) = \{1, ab\}$. Also, $N^\wedge(S)^* = \{b, ab\}, H^\wedge(S)^* = \{a, ab\}, (H \cap N)^\wedge(S)^* = \{ab\}$. Clearly, $\overline{\Gamma}''_{H\cap N} \nsupseteq \overline{\Gamma}''_H \cap \overline{\Gamma}''_N$.

4. Main results

The following theorems are pseudo-Cayley version of Theorems 2.4-2.7 in [7]. One can easily verify them.

Theorem 4.1. Let $PCay(R, S_1)$ and $PCay(R, S_2)$ be pseudo-Cayley graphs, where R is a subset of a group G. Then

(1) $PCay(R, S_1) \cup PCay(R, S_2) = PCay(R, S_1 \cup S_2)$,

(2) $PCay(R, S_1) \cap PCay(R, S_2) = PCay(R, S_1 \cap S_2)$.

Theorem 4.2. Let $PCay(R_1, S)$ and $PCay(R_2, S)$ be pseudo-Cayley graphs, where R_1 and R_2 are subsets of a group G. Then

(1) $PCay(R_1, S) \cup PCay(R_2, S) = PCay(R_1 \cup R_2, S)$,

(2) $PCay(R_1, S) \cap PCay(R_2, S) = PCay(R_1 \cap R_2, S)$.

Theorem 4.3. Let $PCay(R_1, S_1)$ and $PCay(R_2, S_2)$ be pseudo-Cayley graphs, where R_1 and R_2 are subsets of a group G. Then

$PCay(R_1, S_1) \cap PCay(R_2, S_2) = PCay(R_1 \cap R_2, S_1 \cap S_2)$.

Theorem 4.4. Let $PCay(R, S_1)$, $PCay(R, S_2)$, $PCay(R_1, S)$ and $PCay(R_2, S)$ be pseudo-Cayley graphs, where R, R_1 and R_2 are subsets of a group G. Then

(1) $PCay(R, S_1) \subseteq PCay(R, S_2)$ if and only if $S_1 \subseteq S_2$,

(2) $PCay(R_1, S) \subseteq PCay(R_2, S)$ if and only if $R_1 \subseteq R_2$.

The following theorem is the modified version of Theorem 4.6 in [7].

Theorem 4.5. Let N and H be normal subgroups of a group G and let $\Gamma = Cay(G, S)$ be a Cayley graph. Then

(1) $\underline{\Gamma}_{H\cap N} \supseteq \underline{\Gamma}_H \cup \underline{\Gamma}_N \supseteq \underline{\Gamma}_H \cap \underline{\Gamma}_N$,

(2) $\overline{\Gamma}_{H\cap N} \subseteq \overline{\Gamma}_H \cap \overline{\Gamma}_N \subseteq \overline{\Gamma}_H \cup \overline{\Gamma}_N$.

Proof.

(1) By Proposition 2.1(1), $(H \cap N)_-(S) \supseteq H_-(S) \cup N_-(S) \supseteq H_-(S) \cap N_-(S)$. Now, ([7], Theorem 2.7(1)) implies that

$$Cay(G, (H \cap N)_-(S)) \supseteq Cay(G, H_-(S) \cup N_-(S)) \supseteq Cay(G, H_-(S) \cap N_-(S)).$$

On the other hand, from ([7], Theorem 2.4), we have

$$\begin{aligned}\mathsf{Cay}(G, H_-(S) \cup N_-(S)) &= \mathsf{Cay}(G, H_-(S)) \cup \mathsf{Cay}(G, N_-(S)),\\ \mathsf{Cay}(G, H_-(S) \cap N_-(S)) &= \mathsf{Cay}(G, H_-(S)) \cap \mathsf{Cay}(G, N_-(S)).\end{aligned}$$

Therefore $\underline{\Gamma}_{H\cap N} \supseteq \underline{\Gamma}_H \cup \underline{\Gamma}_N \supseteq \underline{\Gamma}_H \cap \underline{\Gamma}_N$.

(2) By Proposition 2.1(2), ([7], Theorem 2.7(1)) and ([7], Theorem 2.4), the proof is similar to (1). □

The converse of some statements of Theorem 4.5 is not necessarily true. For example, let $G = D_8, N = \{1, a^2\}, H = \{1, a, a^2, a^3\}, S = \{b, a^2b\}$ and $\Gamma = \mathsf{Cay}(G, S)$. Then $N_-(S) = \{b, a^2b\}, H_-(S) = \emptyset$ and $(H \cap N)_-(S) = \{b, a^2b\}$. So $\underline{\Gamma}_{H\cap N} = \underline{\Gamma}_H \cup \underline{\Gamma}_N \nsubseteq \underline{\Gamma}_H \cap \underline{\Gamma}_N$. Also, by Example 3.1, one can easily see that $\underline{\Gamma}_{H\cap N} \nsubseteq \underline{\Gamma}_H \cup \underline{\Gamma}_N$ and $\overline{\Gamma}_{H\cap N} \nsupseteq \overline{\Gamma}_H \cap \overline{\Gamma}_N$.

The modified version of Theorem 5.6 in [7] is as follows:

Theorem 4.6. Let N and H be normal subgroups of a group G and let $\Gamma = \mathsf{PCay}(R, S)$ be a pseudo-Cayley graph, where R is a subset of G. Then

(1) $\underline{\Gamma}'_{H\cap N} \supseteq \underline{\Gamma}'_H \cup \underline{\Gamma}'_N \supseteq \underline{\Gamma}'_H \cap \underline{\Gamma}'_N$,

(2) $\overline{\Gamma}'_{H\cap N} \subseteq \overline{\Gamma}'_H \cap \overline{\Gamma}'_N \subseteq \overline{\Gamma}'_H \cup \overline{\Gamma}'_N$.

Proof.

(1) By Proposition 2.1(1), we have

$$(H \cap N)_-(R) \supseteq H_-(R) \cup N_-(R). \tag{4.1}$$

Also,

$$S \cap \big((H \cap N)_-(R)\big) \supseteq S \cap \big(H_-(R) \cup N_-(R)\big) = \big(S \cap H_-(R)\big) \cup \big(S \cap N_-(R)\big). \tag{4.2}$$

Note that

$$\begin{aligned}&\mathsf{PCay}\big((H \cap N)_-(R), S \cap ((H \cap N)_-(R))\big)\\ &\supseteq \mathsf{PCay}\big(H_-(R) \cup N_-(R), S \cap ((H \cap N)_-(R))\big) && \text{by (4.1) and Theorem 4.4(2)}\\ &\supseteq \mathsf{PCay}\big(H_-(R) \cup N_-(R), (S \cap H_-(R)) \cup (S \cap N_-(R))\big) && \text{by (4.2) and Theorem 4.4(1)}\\ &= \mathsf{PCay}\big(H_-(R) \cup N_-(R), S \cap H_-(R)\big) && \text{by Theorem 4.1(1)}\\ &\cup \mathsf{PCay}\big(H_-(R) \cup N_-(R), S \cap N_-(R)\big)\\ &\supseteq \mathsf{PCay}\big(H_-(R), S \cap H_-(R)\big) \cup \mathsf{PCay}\big(N_-(R), S \cap N_-(R)\big). && \text{by Theorem 4.4(2)}\end{aligned}$$

So,

$$\begin{aligned}&\mathsf{PCay}\big((H \cap N)_-(R), S \cap ((H \cap N)_-(R))\big)\\ &\qquad \supseteq \mathsf{PCay}\big(H_-(R), S \cap H_-(R)\big) \cup \mathsf{PCay}\big(N_-(R), S \cap N_-(R)\big).\end{aligned}$$

It is obvious that

$$\begin{aligned}&\mathsf{PCay}\big(H_-(R), S \cap H_-(R)\big) \cup \mathsf{PCay}\big(N_-(R), S \cap N_-(R)\big)\\ &\qquad \supseteq \mathsf{PCay}\big(H_-(R), S \cap H_-(R)\big) \cap \mathsf{PCay}\big(N_-(R), S \cap N_-(R)\big).\end{aligned}$$

Therefore $\underline{\Gamma}'_{H\cap N} \supseteq \underline{\Gamma}'_H \cup \underline{\Gamma}'_N \supseteq \underline{\Gamma}'_H \cap \underline{\Gamma}'_N$.

(2) By Proposition 2.1(2), Theorem 4.4 and Theorem 4.2(2), the proof is similar to (1). □

The converse of some statements of Theorem 4.6 does not hold in general. For example, let $G = \mathbb{Z}_{12}, R = \{0, 2, 4, 5, 6, 8, 10, 11\}, N = \{0, 4, 8\}, H = \{0, 6\}, S = \{6\}$ and $\Gamma = \mathsf{PCay}(R, S)$. Then $N_-(R) = \{0, 2, 4, 6, 8, 10\}, H_-(R) = \{0, 2, 4, 5, 6, 8, 10, 11\}$ and $(H \cap N)_-(R) = \{0, 2, 4, 5, 6, 8, 10, 11\}$. Also, $S \cap N_-(R) = S \cap H_-(R) = S \cap \big((H \cap N)_-(R)\big) = \{6\}$. Thus $\underline{\Gamma}'_{H\cap N} = \underline{\Gamma}'_H \cup \underline{\Gamma}'_N \nsubseteq \underline{\Gamma}'_H \cap \underline{\Gamma}'_N$. Also, by Example 3.2, it is easy to see that $\underline{\Gamma}'_{H\cap N} \nsubseteq \underline{\Gamma}'_H \cup \underline{\Gamma}'_N$ and $\overline{\Gamma}'_{H\cap N} \nsupseteq \overline{\Gamma}'_H \cap \overline{\Gamma}'_N$.

The following theorem is the modified version of Theorem 6.6 in [7].

Theorem 4.7. Let N and H be normal subgroups of a group G and let $\Gamma = PCay(R, S)$ be a pseudo-Cayley graph, where R is a subset of G. Then

(1) $\underline{\Gamma}''_{H\cap N} \supseteq \underline{\Gamma}''_{H} \cup \underline{\Gamma}''_{N} \supseteq \underline{\Gamma}''_{H} \cap \underline{\Gamma}''_{N}$,

(2) $\overline{\Gamma}''_{H\cap N} \subseteq \overline{\Gamma}''_{H} \cap \overline{\Gamma}''_{N} \subseteq \overline{\Gamma}''_{H} \cup \overline{\Gamma}''_{N}$.

Proof.

(1) By Proposition 2.1(1), we have

$$(H \cap N)_{-}(R) \supseteq H_{-}(R) \cup N_{-}(R), \tag{4.3}$$

$$(H \cap N)_{-}(S) \supseteq H_{-}(S) \cup N_{-}(S). \tag{4.4}$$

Note that

$$\begin{aligned}
&PCay\big((H\cap N)_{-}(R), (H\cap N)_{-}(S)\big)\\
&\quad \supseteq PCay\big(H_{-}(R) \cup N_{-}(R), (H\cap N)_{-}(S)\big) && \text{by (4.3) and Theorem 4.4(2)}\\
&\quad = PCay\big(H_{-}(R), (H\cap N)_{-}(S)\big) && \text{by Theorem 4.2(1)}\\
&\quad \cup PCay\big(N_{-}(R), (H\cap N)_{-}(S)\big)\\
&\quad \supseteq PCay\big(H_{-}(R), H_{-}(S) \cup N_{-}(S)\big) && \text{by (4.4) and Theorem 4.4(1)}\\
&\quad \cup PCay\big(N_{-}(R), H_{-}(S) \cup N_{-}(S)\big)\\
&\quad \supseteq PCay\big(H_{-}(R), H_{-}(S)\big) \cup PCay\big(N_{-}(R), N_{-}(S)\big). && \text{by Theorem 4.4(1)}
\end{aligned}$$

Thus

$$\begin{aligned}
&PCay\big((H\cap N)_{-}(R), (H\cap N)_{-}(S)\big)\\
&\quad \supseteq PCay\big(H_{-}(R), H_{-}(S)\big) \cup PCay\big(N_{-}(R), N_{-}(S)\big).
\end{aligned}$$

It is clear that

$$\begin{aligned}
&PCay\big(H_{-}(R), H_{-}(S)\big) \cup PCay\big(N_{-}(R), N_{-}(S)\big)\\
&\quad \supseteq PCay\big(H_{-}(R), H_{-}(S)\big) \cap PCay\big(N_{-}(R), N_{-}(S)\big).
\end{aligned}$$

Therefore $\underline{\Gamma}''_{H\cap N} \supseteq \underline{\Gamma}''_{H} \cup \underline{\Gamma}''_{N} \supseteq \underline{\Gamma}''_{H} \cap \underline{\Gamma}''_{N}$.

(2) By Proposition 2.1(2) and Theorem 4.4 and Theorem 4.2(2), the proof is similar to (1). □

By Example 3.3, one can see that $\underline{\Gamma}''_{H\cap N} \not\subseteq \underline{\Gamma}''_{H} \cup \underline{\Gamma}''_{N} \not\subseteq \underline{\Gamma}''_{H} \cap \underline{\Gamma}''_{N}$ and $\overline{\Gamma}''_{H\cap N} \not\supseteq \overline{\Gamma}''_{H} \cap \overline{\Gamma}''_{N}$. So, obviously, the converse of some statements of Theorem 4.7 is not true.

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